2.5 5th (P.200) #11 15 25 33 37 39 43 49 57 60 65 67 { graph
else only x^2 +

[Quadratic]

2.6 6th (P.229) #16 20 30 40 50 60 69 76

2.6 5th Modeling (P.210) #10 13 17 21 25 27 32 35

(P.213) 6th Focus

2.2 P.220
(a) x + y = 19
   x y as big
   f(x) = x(x-9)
   maximize
   \( x = 9.5 \)
   \( y = 10.5 \)
   max =

(b) \( 2x + y = 2400 \)
   \( A(x) = xy = x(2400-2x) \)
   \( x = 600 \)
   \( y = 1200 \)
   \( A = 720000 \)

2.2 P.213
(a) \( T(x) = \frac{d}{2} + \frac{x}{5} \)
   \( = \sqrt{\frac{d-x^2+2^2}{2}} + \frac{x}{5} \)

(b) max T

7.7 Stadium Revenue
\[ R(x) = xN(x) \]
revenue price
\[ N(10) = 27,000 \]
\[ AN = 3000 \]
\[ \frac{AN}{Ax} = \frac{-2}{1} \]
\[ N(x) = -3000(x-10) + 27,000 \]

7.8 \( R(x) = N(x)(x-10) \)
\[ \text{cost $6 to make} \]
20 /wk at price $10
\[ N(10) = 20 \]
\[ AN = 3000 \]
\[ \frac{AN}{Ax} = \frac{-2}{1} \]
\[ N(x) = -2(x-10) + 20 \]
2.1 (p. 155) # 3, 8, 11, 14, 15, 23, 27, 30 & f'(a)
   47, 51, 52, 57, 69, 71
2.2 (p. 167) # 17, 22, 23, 24, 25, 33, 46, 47, 49, 53, 55, 57, 63, 65, 67, 69, 71, 89 (try int fn.)
2.3 (p. 179) # 3, 9, 19, 27 & instantaneous, 29, 35, 37, 39
2.4 (p. 200) # 11, 15, 25, 33, 37, 39, 43, 49, 57, 65, 67
2.6 (p. 210) # 10, 13, 17, 21, 25, 27, 32, 35.
2.4 (p. 199) # 1–47 odd; 53–57 odd; 61–73 odd; 75–77 every
2.7 (p. 219) # 3, 5, 9, 10, 12, 16, 20, 21, 25, 27, 37, 38, 39, 40, 43, 47, 49, 51–54, 61, 62, 64, 65, 66, 67
   Bonus: 4, 10, 27, 38, 40, 62
2.8 (p. 230) # 4, 6, 7, 10, 13, 18, 19, 30, 38, 40, 46, 47, 53, 67, 70
   Bonus: 83.
   cw: 14, 20, 29, 54, 68
2.1 what is a function?
• See SAT pg. 1

ex1 machine diagram
\[ f(x) = x^2 \]
\[ f: x \rightarrow x^2 \]
\[ \sqrt{5} \rightarrow 5 \]
\[ \sqrt{5} \rightarrow \sqrt{5} \rightarrow 5 \]

ex2 \[ f(x) = 3x^2 + x - 5 \]
\[ f(\frac{3}{2}) = 3 \cdot \frac{3}{4} + \frac{3}{2} - 5 = -\frac{15}{4} \]

ex3 Piecewise-defined function
\[ y = \begin{cases} 2x & x < 1 \\ x+1 & x \geq 1 \end{cases} \]

2.2 Function Graphs
ex2 Graphing calc
\[ y = x^n \quad n = 2, 4, 6 \]
\[ [-2, 2] \times [-1, 3] \]

2.2 Function Graphs
ex2 Graphing calc
\[ y = x^n \quad n = 1, 3, 5 \]
\[ [1, 2] \times [-1, 3] \]
2. Sketch each function and fill in the blanks.

### Library of Functions

<table>
<thead>
<tr>
<th>Name</th>
<th>Function</th>
<th>Sketch</th>
<th>Domain</th>
<th>Range</th>
<th>One point on curve</th>
<th>Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>cubic poly</td>
<td>(y = x^3)</td>
<td><img src="image" alt="Sketch of cubic poly" /></td>
<td>(\mathbb{R})</td>
<td>(\mathbb{R})</td>
<td>((0,0))</td>
<td>none</td>
</tr>
<tr>
<td>quadratic parabola</td>
<td>(y = x^2)</td>
<td><img src="image" alt="Sketch of quadratic parabola" /></td>
<td>(\mathbb{R})</td>
<td>({y \geq 0})</td>
<td>((0,0))</td>
<td>none</td>
</tr>
<tr>
<td>cubic rational</td>
<td>(y = \frac{1}{x})</td>
<td><img src="image" alt="Sketch of cubic rational" /></td>
<td>(\mathbb{R}, {x \neq 0})</td>
<td>({y \neq 0})</td>
<td>((1,1))</td>
<td>(x = 0, y = 0)</td>
</tr>
<tr>
<td>exponential growth</td>
<td>(y = e^x)</td>
<td><img src="image" alt="Sketch of exponential growth" /></td>
<td>(\mathbb{R})</td>
<td>({y &gt; 0})</td>
<td>((0,1))</td>
<td>none</td>
</tr>
<tr>
<td>exponential decay</td>
<td>(y = e^{-x})</td>
<td><img src="image" alt="Sketch of exponential decay" /></td>
<td>(\mathbb{R})</td>
<td>({y &gt; 0})</td>
<td>((0,1))</td>
<td>none</td>
</tr>
<tr>
<td>square root</td>
<td>(y = \sqrt{x})</td>
<td><img src="image" alt="Sketch of square root" /></td>
<td>({x \geq 0})</td>
<td>({y \geq 0})</td>
<td>((0,0))</td>
<td>none</td>
</tr>
<tr>
<td>cubed root</td>
<td>(y = \sqrt[3]{x})</td>
<td><img src="image" alt="Sketch of cubed root" /></td>
<td>(\mathbb{R})</td>
<td>({y \geq 0})</td>
<td>((0,0))</td>
<td>none</td>
</tr>
<tr>
<td>logarithmic growth</td>
<td>(y = \log x)</td>
<td><img src="image" alt="Sketch of logarithmic growth" /></td>
<td>({x &gt; 0})</td>
<td>(\mathbb{R})</td>
<td>((1,0))</td>
<td>(x = 0)</td>
</tr>
<tr>
<td>logarithmic decay</td>
<td>(y = \log_a x)</td>
<td><img src="image" alt="Sketch of logarithmic decay" /></td>
<td>({x &gt; 0})</td>
<td>({y \geq 0})</td>
<td>((1,0))</td>
<td>(x = 0)</td>
</tr>
</tbody>
</table>

#### Function Symmetry

- **Odd**: \(f(-x) = -f(x)\)
- **Even**: \(f(-x) = f(x)\)

- \(\text{odd} + \text{odd} = \text{odd}\)
- \(\text{even} + \text{odd} = \text{odd}\)
- \(\text{even} \times \text{even} = \text{even}\)
- \(\text{odd} \times \text{odd} = \text{odd}\)
- \(\text{odd} + \text{even} = \text{neither}\)
- \(\text{even} \times \text{odd} = \text{even}\)

#### Trigonometric Table

<table>
<thead>
<tr>
<th>(\theta) in Radians</th>
<th>0</th>
<th>(\pi/6)</th>
<th>(\pi/4)</th>
<th>(\pi/3)</th>
<th>(\pi/2)</th>
<th>(\pi)</th>
<th>(3\pi/2)</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin\theta)</td>
<td>0</td>
<td>(1/2)</td>
<td>(\sqrt{2}/2)</td>
<td>(\sqrt{3}/2)</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>(\cos\theta)</td>
<td>1</td>
<td>(\sqrt{2}/2)</td>
<td>(\sqrt{3}/2)</td>
<td>(1/2)</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(\tan\theta)</td>
<td>0</td>
<td>(\sqrt{3}/3)</td>
<td>1</td>
<td>(\sqrt{3})</td>
<td>dne</td>
<td>0</td>
<td>dne</td>
<td>0</td>
</tr>
<tr>
<td>(\sec\theta)</td>
<td>1</td>
<td>(2/\sqrt{3})</td>
<td>(\sqrt{2})</td>
<td>2</td>
<td>dne</td>
<td>-1</td>
<td>dne</td>
<td>1</td>
</tr>
</tbody>
</table>
(3) Sketch the graph of $y = 3 \sin(x-\pi)$ without using a calculator.

(4) Sketch the graph of $y = x + 3 + \frac{3}{x}$ without using a calculator.

(5) Sketch the graph of $y = x^3 + 2$ without using a calculator.

(6) Sketch the graph of $y = |x-3| + 2$ without using a calculator.

(7) Sketch the graph of $y = f(x) = \frac{x}{x^2 - 3}$ without using a calculator.

(8) Sketch the graph of $y = f(x) = \frac{x}{x-3}$ without using a calculator.

(9) Sketch the graph of $y = f(x) = \frac{x}{x^2 - 3}$ without using a calculator.

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(59) Sketch the graph of $y = f(x) = \frac{x}{x^2 - 3}$ without using a calculator.

(60) Sketch the graph of $y = f(x) = \frac{x}{x-3}$ without using a calculator.
2.2 \[ f(x) = \begin{cases} x^2 & x \leq 1 \\ 2x+1 & x > 1 \end{cases} \]

which is bigger? \[ x^2 > x^2, x^2 > x^2 \]

CALC
\[ y_1 = (x \leq 1)x^2 + (x > 1)(2x+1) \]

2nd (test)

\[ f(x) = \text{int}(x) = \text{floor}(x) \]

- \( f(1) = 1 \)
- \( f(1.5) = 1 \)
- \( f(7.8) = 7 \)
- \( f(-2.1) = -3 \)

MATH NUM
\[ \text{int}(x) \]

Describe the motion:
- \( 0 \leq x < 0 \)
- \( 0 \leq x < 1 \)
- \( 1 \leq x < 2 \)
- \( 2 \leq x < 3 \)

C(t) = cost of phone call for t minutes, \( v_{\text{inst}}(t) = 2 \)

- 69¢ 1st minute
- 58¢ each extra minute
- \( f(t) = 0.69 + 0.58(t-1) \)

\[ v_{\text{avg}} = \frac{s(t) - s(t_0)}{t - t_0} \]

\[ v_{\text{avg}} = \lim_{t \to t_0} \frac{s(t) - s(t_0)}{t - t_0} \]

- \( v(t) = \lim_{t \to t_0} \frac{s(t) - s(t_0)}{t - t_0} = v_{\text{inst}}(t_0) \)

Motion:
- Position
- Instantaneous speed
- Speed = how fast position changes wrt time
- Average speed = slope of secant line
- Instantaneous speed = slope of tangent line

\[ v(t) = \frac{\Delta y}{\Delta t} \]

\[ u(t) = \lim_{t \to 0} \frac{\Delta y}{\Delta t} = \lim_{t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t} \]

\[ v(t) = \lim_{t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t} \]

\[ v(t) = \lim_{t \to 0} \frac{y(u) - y(t)}{t - u} \]

- \( v(t) = \lim_{t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t} \)

\[ v_{\text{avg}} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} \]

\[ v_{\text{avg}} = \frac{s(3) - s(2)}{3 - 2} = \frac{140 - 75}{1} = 65 \text{ mi/h} \]

\[ u_{\text{avg}} = \frac{s(4) - s(1)}{4 - 1} = \frac{300 - 50 \text{ mi}}{3} = 50 \text{ mi/h} \]

\[ v(t) = \text{slope of tangent line} = 50 \text{ mph} \]

\[ f(a) = \text{slope of secant line} \]

- Physics example on rates
- Max:
- Min:
- Local maximum
- Local minimum
- Max:
- Min:
- Local maximum
- Local minimum

\[ f(x) \leq f(a) \forall x \in I \]

- Local (relative) max:
- f(a) is a
- \( f(b) \) is a local/relative minu on I:
- \( f(x) \geq f(b) \forall x \in I \)

- Max:
- Min:
- Local max & Min
\[ f(x) = (x - 3)^2 \]

Avg rate of change on \([4,7]\):

\[ \frac{4f}{dx} = \frac{f(7) - f(4)}{7 - 4} = \frac{161}{3} = 5 \]

slope of secant line on \([3,7]\):

\[ \frac{f(7) - f(3)}{7 - 3} \]

Avg rate of change on \([a, a+h]\) is 3:

\[ \text{slope rate of change at } x = a \]

\[ f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \]

\[ = \lim_{h \to 0} \frac{3a + 3h - 5 - 3a + 5}{h} = \lim_{h \to 0} \frac{3h}{h} = 3 \]

\[ \text{avg = inst for a line, secant slope = tangent slope} \]

2.3 Strictly Monotonically Increasing:

\[ f'(u) > 0 \text{ if } u \in [a, b], \quad f'(u) < 0 \text{ if } u \in (a, b) \]

Max or Min & Where?

Max is at \( x = \frac{1}{2} \), max value is \( \frac{9}{4} \)
2.3

- Upper Bound
  - Least Upper Bound \( \{ \text{maximum, achieve} \} \) \( f(x) \leq M \forall x \in I \)
  - Greatest Lower Bound \( \{ \text{minimum, infimum} \} \) \( f(x) \geq M \forall x \in I \)

- Saddle Point
  - both a max & min

Function:
- vertical line test: \( \Box \)
- horizontal line test: \( \Box \)

Diagram:
- \( x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \)
- \( x_1 = x_2 \Rightarrow f(x_1) = f(x_2) \)
- \( f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \)
- \( f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2 \)
- \( x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \)

(2.7) one-to-one function: \( x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \)

Prologue example
- Prologue P4 amoeba
- 76 #63 Bob & Jim

Contrapositive
- \( \Box \Rightarrow R \)
- \( R \Rightarrow C \)
- \( R \subseteq C \)

P2
ex3 \[ M(s) = -\frac{1}{28}s^3 + 3s - 31, \quad 15 \leq s \leq 70 \]

a) Max gas mileage = \( \frac{32 \text{ mi/gal}}{42 \text{ mph}} \)
when speed = \( \frac{3}{2} \text{ mph} \)
\[ \frac{-\frac{3}{2}}{\frac{1}{28}} = 14 \times 3 = 42 \]
max mileage = \( M(42) = \frac{32}{42^2} + 3.42 - 31 \)
b) Get in standard form; from \[ M(s) = -\frac{1}{28}(s - 42)^2 + 32 \]
c) Get in standard form by completing square \[ M(s) = -\frac{1}{28}(s^2 - 84s + 42^2) - 31 + \frac{42^2}{28} = 3.2 \]
d) factored form
\[ r_1, r_2 = -3 \pm \sqrt{9 - 4\left(\frac{28}{2}\right)(31)} = 12.07 \]
\[ M(s) = -\frac{1}{28}(s - r_1)(s - r_2) \]

SHOW CALC Check

\[ a = 0, b = 3, c = -31 \]
ax \^2 + bx + c = 0 \rightarrow ax \left( x + \frac{b}{a} \right) \leftarrow c = 0 \rightarrow \text{use factored form}

ex7 \[ \text{max value of} \]
\[ f(t) = -0.113t^3 + 0.681t^2 + 0.198t + 99.1 \]
\[ \lambda(t) = \begin{cases} -0.113t^3 + 0.681t^2 + 0.198t + 99.1 \quad \text{for} \quad 0 \leq t \leq 14 \\ \end{cases} \]
\[ \lambda(t) = \begin{cases} -0.113t^3 + 0.681t^2 + 0.198t + 99.1 \quad \text{for} \quad 14 < t \leq 15 \\ \end{cases} \]
\[ \lambda(t) = \begin{cases} -0.113t^3 + 0.681t^2 + 0.198t + 99.1 \quad \text{for} \quad t > 15 \end{cases} \]

(ex 2.6) Model
\[ f(t) \text{ is an algebraic model from geometry} \]
\[ V(r) = \frac{1}{3} \pi r^2 \]

Focus
\[ \text{Modeling from data} \quad \text{linear regression} \]

Later

\[ a = -10 \text{ m/s}^2 \]

\[ \text{140 feet of fencing} \]
\[ 2x + 2x = 140 \]
\[ x = 35 \]
\[ A = xl \]
\[ A(x) = x(70-x) \]

a) function for area
\[ l = 70-x \]
\[ \text{CALC} \]

b) what range of widths make area \( \geq 825 \text{ ft}^2 \)
\[ x(70-x) \geq 825 \]
\[ x^2 - 70x + 825 \leq 0 \]
\[ (x-15)(x-55) \leq 0 \]
\[ 15 \leq x \leq 55 \]
\[ \text{no} \text{ for area} \geq 1250 \text{ ft}^2 \]
\[ x^2 - 70x + 1250 = 0 \]
\[ x \notin \mathbb{R} \]

Note: \( ax^2 + bx + c \rightarrow ax(x + \frac{b}{a}) \rightarrow c = 0 \rightarrow \text{use factored form} \]
Practice Pg 211
Area problem : 23
Revenue/Line word prob

minimize time

what radius minimizes the amount of metal in a can that holds 1 L of water?

Objective: surface area

\[ A(r) = 2\pi r^2 + 2\pi rh \]

\[ \tau r^2 h = 1000 \text{ cm}^3 \]

\[ h = \frac{1000}{\tau r^2} \]

\[ A(r) = 2\pi r^2 + 2\pi \left(\frac{1000}{\tau r^2}\right) \]

\[ A(r) = \frac{2\pi r^2}{r^2} + \frac{2000}{r} \]

\[ A(r) = \frac{2\pi r^2}{r^2} + \frac{2000}{r} \]

\[ \text{minimize at } r = \frac{5.4}{cm} \]

\[ A = 554 \text{ cm}^2 \]

\[ \text{For each point below 44, the scaled score decreases by 10 points.} \]

\[ S(R) = 10(R-44) + 800 \]

\[ S(44) = 800 \]

SAT 50 Questions
R \geq 44 \Rightarrow S = 800

A hockey team plays in an arena with a seating capacity of 15000 ppl with ticket price $14, avg attendance at games is 9500
A survey shows for each dollar the ticket price is lowered, avg attendance increases by 600.
\[ A(14) = 9500 \]
\[ \frac{dA}{dt} = -1000 \]

\[ t = \frac{11.75}{14} \]
**Transformations**

- **Symmetry even/odd SAT**
  - Even: $f(x) = f(-x)$
  - Odd: $f(-x) = -f(x)$

- **Translations**
  - $g(x) = f(x) + k$
  - $g(x) = f(x) - k$
  - $g(x) = f(x+a)$

- **Reflections**
  - $g(x) = -f(x)$
  - $g(x) = f(-x)$
  - $g(x) = -f(-x)$

- **Scaling**
  - $g(x) = af(x)$
  - $g(x) = f(ax)$
  - $g(x) = f(x/a)$

- **Absolute Value**
  - $|f(x)|$

- **Practice Examples**
  - $f(x) = x^3$
  - $g(x) = (x+3)^3$
  - $h(x) = (x-2)^3$

- **Other Examples**
  - $g(x) = f(x+3)$
  - $g(x) = f(x-3)$
  - $g(x) = f(-x)$
  - $g(x) = -f(x)$

- **Notes**
  - $(2x)^3 = 8x^3$
  - $(x/3)^2 = x/9$
2.5 6th ed: Symmetry even odd see SAT p4-5

* (2,4) Transformations * Libary
- see SAT pg.5
- make page for chart (tell students)

3.1 Translations

ex1 \( f(x) = x^2 \) write in HW
a) \( g(x) = x^2 + 3 = f(x) + 3 \)
b) \( h(x) = x^2 - 2 = f(x) - 2 \)

ex2 \( f(x) = x^3 - 9x = x(x-3)(x+3) \) write in HW
a) \( g(x) = x^3 - 9x + 10 = f(x) + 10 \)
b) \( h(x) = x^3 - 9x - 20 = f(x) - 20 \)

ex3 \( f(x) = x^2 \) write in HW.

a) \( g(x) = (x+4)^2 = f(x+4) \)
b) \( h(x) = (x-2)^2 = f(x-2) \)

*ex4 Combining \( p(x) = \sqrt{x} \)
\( f(x) = \sqrt{x-3} + 4 \)
write \( p(x-3) + 4 \)
write in HW

\( r(x) = x^3 \)
a) move right 2 \( (x-2)^3 = h \)
flip down \( (x-2)^3 = -h = -g \)
Stretch \$3 \( 3 \cdot 3 = 3h \)
\(-3(x-3)^3 - 3 = 3l \)

2.2 Reflections - See SAT pg 5

a) \( f(x) = -x^2 \)
b) \( g(x) = \sqrt{-x} \)

c) \( r(-x) \)

3.3 Stretch/Shrink Scaling - See SAT pg 5

ex6 \( f(x) = x^2 + 4x^2 \)
a) \( g(x) = 3x^2 \)
b) \( h(x) = \frac{1}{3} x^2 \)

ex7 \( f(x) = 1 - 2(x - 3)^2 \)

Is any order OK? No!
Stretch before translate
Vertically opposite

Parent fn, \( r(x) = x^2 \)

As \( (3x-1) \)

\( \text{OK} \)
\( r(x) \)
move right
stretch & flip
up 1

\( \text{OK} \)
\( r(x) \)
move right
stretch before translate

Thinking backwards
Horiz Stretch/Shrink

HW: Show scale

1-periodic

\[ f(-x) = f(x) \]

symm. about y-axis

\[ f(-x) = -f(x) \]

symm. about origin

-odd

\[ f(-x) = f(x) \]

ev

\[ f(-x) = -f(x) \]

-odd

\[ f(-x) = f(x) \]

oddd + odd

\[ h(-x) = 2x - x^2 \]

neither

\[ h(-x) = -2x - x^2 \]

\[ h(-x) = -h(x) \]

\[ h(-x) = -h(x) \]

Class Practice - d give answers

Pg 190 *use graph paper

1. \( f(x) = 4x - x^2 \)

\[ f(x) = 4x - x^2 \]

\[ x(4-x) \]

next: Multiple horizontal transformations

1) factor out the const in front of \( x \)

2) Outside in: (Scale before translate in each direction)

\[ y = f(-Bx + C) = f\left(-B\left(x - \frac{C}{B}\right)\right) \]

\[ f(x + C) = g(x) \]

\[ (x+1)^2 \]

\[ f(-Bx) = g(x) \]

\[ (2x+1)^2 \]

left, then shrunk
Combining Transformations

Like Quiz

$r(x) = x^3$

move right 2

$r(x-2) = (x-2)^3$

flip down 1

$-r(x-2) = -(x-2)^3$

vertical stretch $\downarrow 3$

$-3r(x-3) = -3(x-3)^3 = -3\cdot 2^3 = -27$

horizontal stretch $\downarrow 3$

horz-op order

vertical $\frac{3}{2}$

order of ops

$r(x) = x^3$

horizontal & vertical are independent!

Alternatively

$y = A f(B(x-h)) + k$

Horizontal: always opposite arithmetic order of operations

Vertical: always same as order of ops.

Sketch $g(x) = 3e^{-2x+4} + 2$

Like Quiz

$f(x) = e^x$

left 2

$f(x+2) = \sqrt{x+2} + 2$

up 3

$f(x+2) + 3 = \sqrt{x+2} + 3$

flip down

$-f(x+2) + 3 = -\sqrt{x+2} - 3$

$\downarrow 3$

$-f(3x+2) - 3 = -3(3x+2) - 3$

$\downarrow 5$

$-5f(3x+2) - 15 = -5\sqrt{3x+2} - 15$

flip left

$-5f(-3x+2) - 15 = -5\sqrt{-3x+2} - 15$

(see P4 order ok?)

Physics: air drag

$v(t) = v_0(1 - e^{-kt})$

(f = kv)
Sketch \( y = -2 \frac{1}{2} f(x) \) - 2

Sketch \( y = -2 \frac{1}{2} f(-\frac{1}{2}x) - 1 \)

Sketch \( y = -2 \frac{1}{2} f(-\frac{1}{2} (x-1)) - 2 \)

Sketch \( y = -2 \frac{1}{2} f(-\frac{1}{2} (x+2)) - 2 \)

Sketch \( y = -2 \frac{1}{2} f(-\frac{1}{2} (x+1)) - 2 \)

Sketch \( y = -2 \frac{1}{2} f(-\frac{1}{2} (x-2)) - 2 \)

Sketch \( y = -2 \frac{1}{2} f(-\frac{1}{2} (x+2)) + 1 \)

Sketch \( y = -2 \frac{1}{2} f(-2(x-2)) + 1 \)

Sketch \( y = -2 \frac{1}{2} f(-2(x-2)) + 1 \)

Sketch \( y = -2 \frac{1}{2} f(-2(x-2)) + 1 \)

Sketch \( y = -2 \frac{1}{2} f(-2(x-2)) + 1 \)

(See ex7 on p.4)

ex9 p4 odd/even
Algebra of Combining Functions

2.6 \( f \) has domain \( A \) \( \Rightarrow \) \((f \pm g)(x)\) has domain \( A \cap B \) \( g(x) \neq 0 \)

\[ f(x) = \frac{1}{x-2} \]
\[ g(x) = \sqrt{x} \]

\( f + g = \frac{1}{x-2} + \sqrt{x} \) \( \text{Domain: } (0,2) \cup (2, \infty) \)
\( f - g = \frac{1}{x-2} - \sqrt{x} \) \( \text{Domain: } (0,2) \cup (2, \infty) \)

\( fg = \frac{\sqrt{x}}{x-2} \)

\[ \frac{f}{g} = \frac{1}{(x-2)\sqrt{x}} \]
**Composition**

\[(f \circ g)(x) = f(g(x)) \neq (g \circ f)(x)\]

- \(f(x) = \sqrt{x}\)
  - Domain: \(\{x \geq 0\}\)
  - \(g(x) = \sqrt{2-x}\)
  - \(2-x \geq 0\) \(\Rightarrow x \leq 2\)

a) \(f \circ g = \sqrt{2-x}\) \(\quad x \leq 2\) \((-\infty, 2]\)

b) \(g \circ f = \sqrt{2-\sqrt{x}}\)
  - \(x \geq 0\)
  - \(2-\sqrt{x} \leq 2\) \(\Rightarrow \sqrt{x} \leq 4\)
  - \(0 \leq x \leq 16\)

\((g \circ g)(x) = \sqrt{2-\sqrt{2-x}}\)
  - \(x \leq 2\)
  - \(2-x \leq 2\) \(\Rightarrow x \geq 0\)
  - \(0 \leq x \leq 2\)

\[f(x) = \frac{x}{x+1}\]

\[g(x) = x^2\]

\[(f \circ g)(x) = \frac{(x+3)^2}{(x+3)^2 + 1}\]

\[h(x) = x+3\]

\[f'(x) - f(a)\]

\[\frac{f(b) - f(a)}{b-a}\]
**One-to-One Function & Inverse**

**Proof:** Assume \( f(x_1) = f(x_2) \). Then \( x_1 = x_2 \).

**Horizontal Line Test**

Is \( g(x) = x^2 \) one-to-one?

- No!

\[ \begin{align*}
g(-1) &= 1 \\
g(1) &= 1
\end{align*} \]

\[ y \text{ fn of } x \\
\text{not fn of } y \]

**Show a fn is one-to-one**

\[ \begin{align*}
f(x) &= 3x + 4 \\
g(x) &= x^2
\end{align*} \]

**Logic**

- \( f(x_1) = f(x_2) \) \( \Rightarrow \) \( x_1 = x_2 \)
- \( 3x_1 + k = 3x_2 + k \) \( \Rightarrow \) \( x_1 = x_2 \)

Therefore \( f \) is one-to-one.

**Inverse fn property**

\[ \begin{align*}
y &= f(x) = x^3 \\
x &= \sqrt[3]{y}
\end{align*} \]

\( f \) & \( f^{-1} \) undo each other

\[ f(f^{-1}(y)) = y \Longleftrightarrow f \text{ & } f^{-1} \text{ are inverse functions} \]

**Verifying inverses**

\[ \begin{align*}
f(x) &= x^3 \\
g(x) &= x^{\frac{1}{3}}
\end{align*} \]

\[ \begin{align*}
f(g(x)) &= (x^{\frac{1}{3}})^3 = x \\
g(f(x)) &= (x^3)^{\frac{1}{3}} = x
\end{align*} \]

**How to find inverse**

1. \( y = f(x) \)
2. \( x = \text{value for } x \text{ in } f^{-1}(x) \)
3. \( f^{-1}(x) = \frac{y}{x} \text{ dummy variable, } u \rightarrow f(u) \)

**Graph of inverse**

- \( f(x) = \frac{x^2 - 3}{2} \)
- \( a) \text{ Sketch } f \)
- \( b) \text{ Sketch } f^{-1} \)
- \( c) \text{ Find } f^{-1}(x) = x^2 + 2 \)
**Chi Focus**

**Linear Regression (pg. 239)** & prediction

- **Height**
  - **foot-size** best-fit line
  - \( y = mx + b \)
  - \((20\text{ cm}, 158\text{ cm})\)

**Formula in calculus** - See CalcC Bonus Worksheet

\[ \hat{m}, \hat{b} = \arg \min_{m, b} \sum_{i=1}^{n} (y_i - (mx_i + b))^2 \]

- Sum of least squares

\[ \hat{m} = \text{formula in calculus} \]

\[ \hat{b} = \frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) \frac{\sum (x - \bar{x})^2}{\sum (x - \bar{x})^2} \]

- \(-1 \leq r \leq 1\)
- \(r \approx 0\) uncorrelated correlation coefficient \(r\) indicates how good a fit

\[ \hat{r} = 0.98 \]

**HW** answer \(a, b, c, d\) for \(x=\text{foot length}\)

**y = height**

**TI-84**

- **Intro Linear Least Squares - Calculus will derive**
- **Home**, clear A to Z
- **F6** \(\Rightarrow\) **New Prob** clears all (lists, data) grid on
- **Apps** \(\Rightarrow\) data/matrix editor
  - **F2** New variable: tree \(\text{Fg(239)}\)
  - **F1** define
    - **F6** clear column
  - **F2** Plot Setup
    - **F1** define
      - **F3** Scatter
      - **F1** X List \(\Rightarrow\) L3
      - **F1** Y List \(\Rightarrow\) L4
      - **F2** Mark
      - **F0** Home
  - **Stat Plot**
    - **Plot** on
      - **F1** Scatter
      - **F1** X List \(\Rightarrow\) L3, L1
      - **F1** Y List \(\Rightarrow\) L4
      - **F1** Mark
      - **F0** Home

- **Stat → Calc**
  - **LinReg** \((a + bx)\) L3, L4, Y1
    - **VARS** \(\Rightarrow\) Y-Vars (Store)
      - **F1** Y = \( Y1 \) has Line
        - **F2** Window
        - **F5** Trace

**Largest feet: Ibrahim Takaiyullah**

- 38.1 cm Long left foot
- 246 cm (>8 ft) tall

- Solve \(\Phi EE \Rightarrow\) KEY of short cut
  - **F7** → Log
  - **F2** Solve \((x^2 - 1 = 0, x)\)
  - **Graphical**
Graphing Calculator

1. Basic
   \[ y = x^2 \]
   \[ y = x^2 + 2x + 1 \]
   \[ y = 2.0 - 1.4x \]
   \[ y = \pm \sqrt{1 - x^2} \]
   Intersect point

2. Graphing a Circle
   \[ x^2 + y^2 = 1 \]
   \[ y^2 = 1 - x^2 \]
   \[ y_1 = \pm \sqrt{1 - x^2} \]
   \[ y_2 \]
   Fit box or square?

Note - can even write programs for TI Pac Man?

3. Linear Regression - See Ch. 2 notes at end
   - Average height \( \rightarrow \) 1-var STATS
   - Standard deviation

4. Solve \[ x^2 - 5x^2 = 2 - 8 \]
   Ans: zeros, intervals

5. Solve \[ x^2 - 5x + 8 = 2.0 - 1.4x \]
   Ans: zeros, intervals

6. TI-89
   TI-84
   Clear all:
   Vars:
   Clear: Home
   one var: \( \hat{Y} \) Delvar a
   done

\[ \text{Answer: } \begin{cases} x_1 =  \frac{\ln x}{\ln 2} \quad \text{or} \quad \log (x, 2) \end{cases} \]

\[ \text{Matrix: } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ -5 & 2 & 3 \end{bmatrix} \]

\[ \text{det} B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ -5 & 2 & 3 \end{bmatrix} \]

\[ B^T = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ -5 & 2 & 3 \end{bmatrix} \]

\[ B^T = \begin{bmatrix} \frac{x}{2} \\ 3 \\ 1 \end{bmatrix} = \langle 4, -8, 4 \rangle \]
\[ y = \begin{cases} 
\frac{x^2}{2x+1} & x \leq 1 \\
\frac{3}{4} & x > 1 
\end{cases} \]

**CALC**

\[ Y^1_i = (x \leq 1) \cdot 2 + (x > 1)(2x+1) \]

2nd (Test)