Calculus Worksheet for AP Physics C


**Topic #1: Rates of Change**  (Anton 2.1)

3. The accompanying figure shows the position versus time curve for a certain particle moving along a straight line. Estimate each of the following from the graph:
   (a) the average velocity over the interval $0 \leq t \leq 3$
   (b) the values of $t$ at which the instantaneous velocity is zero
   (c) the values of $t$ at which the instantaneous velocity is either a maximum or a minimum
   (d) the instantaneous velocity when $t = 3$ s.

![Graph of position versus time](image)

(Anton 2.2)

23. Match the graphs of the functions shown in (a)–(f) with the graphs of their derivatives in (A)–(F).

![Graphs of functions and derivatives](image)
Topic #2, 5: Memorize the Derivative Formulas

(Anton 2.4) Product Rule

5-20 Find $f'(x)$.

5. $f(x) = (3x^2 + 6)(2x - \frac{1}{2})$
6. $f(x) = (2 - x - 3x^3)(7 + x^5)$
7. $f(x) = (x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})$
8. $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$

Quotient Rule

21-24 Find $\frac{dy}{dx}|_{x=a}$.

21. $y = \frac{2x - 1}{x + 3}$
22. $y = \frac{4x + 1}{x^2 - 5}$

(Anton Ch. 2 Review) Product, Quotient, Chain Rules

29-32 Find $f'(x)$.

29. (a) $f(x) = x^8 - 3\sqrt{x} + 5x^{-3}$
   (b) $f(x) = (2x + 1)^{101}(5x^2 - 7)$
30. (a) $f(x) = \sin x + 2\cos^3 x$

31. (a) $f(x) = \sqrt{3x + 1}(x - 1)^2$
   (b) $f(x) = \left(\frac{3x + 1}{x^2}\right)^3$

(b) $f(x) = \frac{1}{2x + \sin^3 x}$
Topic #3: Memorize the antiderivative formulas (Anton 5.2)

2. In each part, confirm that the stated formula is correct by differentiating.
   (a) \( \int x \sin x \, dx = \sin x - x \cos x + C \)

5–8 Find the derivative and state a corresponding integration formula.

7. \( \frac{d}{dx} [\sin(2\sqrt{x})] \)

11–14 Evaluate each integral by applying Theorem 5.2.3 and Formula 2 in Table 5.2.1 appropriately.

11. \( \int \left[ 5x + \frac{2}{3x^5} \right] \, dx \)

13. \( \int \left[ x^{-3} - 3x^{1/4} + 8x^2 \right] \, dx \)

14. \( \int \left[ \frac{10}{\sqrt[3]{y}} - 3\sqrt{y} + \frac{4}{\sqrt{y}} \right] \, dy \)

11. \( \int \left[ 5x + \frac{2}{3x^5} \right] \, dx \)

13. \( \int \left[ x^{-3} - 3x^{1/4} + 8x^2 \right] \, dx \)

14. \( \int \left[ \frac{10}{\sqrt[3]{y}} - 3\sqrt{y} + \frac{4}{\sqrt{y}} \right] \, dy \)

(Anton 3.2)

1–26 Find \( dy/dx \).

1. \( y = \ln 5x \)

7. \( y = \ln \left( \frac{x}{1 + x^2} \right) \)

19. \( y = \ln(\ln x) \)

25. \( y = \log(\sin^2 x) \)

(Anton 3.3)

15–26 Find \( dy/dx \).

15. \( y = e^{7x} \)

17. \( y = x^3 e^x \)

21. \( \int \left[ \frac{2}{x} + 3e^x \right] \, dx \)
Topic #4: Definite Integral as Area and the Fundamental Theorem  
(Anton 5.5)
Draw a picture and use geometric shape formulas.

18. In each part, evaluate the integral, given that
\[ f(x) = \begin{cases} 
2x, & x \leq 1 \\
2, & x > 1 
\end{cases} \]
(a) \( \int_0^1 f(x) \, dx \) 
(b) \( \int_{-1}^1 f(x) \, dx \) 
(c) \( \int_{-1}^0 f(x) \, dx \) 
(d) \( \int_{1/2}^1 f(x) \, dx \) 

(Anton 5.6)
Use the Fundamental Theorem of Calculus to find the following areas:

21. Find \( \int_{-1}^2 [f(x) + 2g(x)] \, dx \) if
\[
\int_{-1}^2 f(x) \, dx = 5 \quad \text{and} \quad \int_{-1}^2 g(x) \, dx = -3
\]

17. \( \int_4^9 2x\sqrt{x} \, dx \)
19. \( \int_{-\pi/2}^{\pi/2} \sin \theta \, d\theta \)
21. \( \int_{-\pi/4}^{\pi/4} \cos x \, dx \)
23. \( \int_{\ln 2}^3 5e^x \, dx \)
24. \( \int_{1/2}^1 \frac{1}{2x} \, dx \)
Topic #6: u-substitution for Integrals.
(Anton 5.3)

Evaluate the integrals using appropriate substitutions.

Show \( u = \), \( du = \) for at least 2 problems. If you become comfortable, you can skip explicitly showing \( u \) and just balance out constants as shown in the Tutorial on page 4.

15. \[ \int (4x - 3)^9 \, dx \]
17. \[ \int \sin 7x \, dx \]
29. \[ \int \frac{x^3}{(5x^4 + 2)^3} \, dx \]
31. \[ \int e^{\sin x} \cos x \, dx \]
33. \[ \int x^2 e^{-2x^3} \, dx \]
35. \[ \int \frac{e^x}{1 + e^{2x}} \, dx \] *bonus
37. \[ \int \frac{\sin(5/x)}{x^2} \, dx \]
39. \[ \int \cos^4 3t \sin 3t \, dt \] *bonus

13. \[ \int_{-2}^{-1} \frac{x}{(x^2 + 2)^3} \, dx \]
15. \[ \int_{\ln 3}^{\ln 3} \frac{e^x}{e^x + 4} \, dx \]
Topic #10: Differential Equations
(Anton 5.2)

Example 6 Solve the initial-value problem
\[
\frac{dy}{dx} = \cos x, \quad y(0) = 1
\]

Obtain an equation with x, y and constants. You do not need to isolate y.

a) Use the indefinite integral method.

b) Use the definite integral method.

Topic #11: Slice, Approximate, Integrate.

A right circular cylinder of radius \( R \) and height \( H \) is completely filled with water. Find the work needed to pump all the water out of the cylinder. Use density \( \rho \) and gravitational constant \( g \).

Hint: slice = circular disc of thickness \( \Delta y \). \( \Delta W = \Delta mgh \). Where \( h \) depends on \( y \).

(Barron’s #3.45)

After a volcano erupts, pieces can be found scattered around the center of the blast. The density of volcano fragments lying \( x \) meters from the point of eruption is given by

\[
N(x) = \frac{2x}{1 + x^{3/2}} \text{ fragments per square meter.}
\]

How many fragments will be found within 20 meters of the point where the volcano exploded? (Set up the definite integral. You can’t evaluate it easily by hand, so use a graphing calculator.)

(A) 13  (B) 278  (C) 556  (D) 712  (E) 4383