11.1 - 11.3 Simple Harmonic Motion / Oscillator

1 Frequency Sinusoidal

1. Condition
   \[ F = -kx \] (elastic region, small amplitude)
   \[ x(t) = A \cos(\omega t + \phi) \]
   *Restoring force
   Stable equilibrium
   displacement ~ sinusoid
   (phase angle "attaching")

2. Show spring example
   \( A = \text{amplitude} \)
   \( T = \text{time for one cycle} = \text{period} \)
   \( \omega = 2\pi f \) (rad/s) = angular frequency

3. Can intuitively see it's sinusoidal

4. \( x(0) = 0 \) equilibrium

5. \( U_{\text{max}} = \frac{1}{2}kA^2 \)

6. \( T = \frac{2\pi}{\omega} \)

7. \( F_x = -kx \)
   \( m \frac{d^2x}{dt^2} = -kx \)

8. \( \omega = \sqrt{\frac{k}{m}} \)

9. Show applies

10. \( T = \frac{2\pi}{\omega} \) (period)

11. \( T, f \) independent of \( A \)

12. \( \frac{m}{k} \), inertia, slower

13. \( k \), stiff, faster

14. \( \omega \), frequency

15. Musical instrument, pendulum clock

16. Show
   \[ v = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}} \]

17. \[ \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2 \]
   \[ v^2 = \frac{k}{m} (A^2 - x^2) = \pm \frac{k}{m} A^2 \sqrt{1 - \frac{x^2}{A^2}} \]
   \[ = \pm v_{\text{max}} \sqrt{1 - \frac{x^2}{A^2}} \]

18. Vertical Spring

19. Use this as \( x = 0 \times \)

20. (See ex to ex 2)
mass on horizontal spring.
where, if any, is \(a=0\)? \[x = \begin{cases} 0 \\ +A \end{cases} \]
a) \(+A\) & +A

(b) \(f = 1.25 \text{ Hz} \Rightarrow 100 \text{ oscillations in } 80 \text{ seconds}
\left[ \frac{5}{4} \text{ sec} \times \frac{20}{20} \right]

c) \text{family of four: } 200 \text{ kg car: } 1200 \text{ kg}

when step in, car spring compressed 3.0 cm,
\[k = ? \quad F = \frac{200 \times 9.8}{0.03} = \frac{6.5 \times 10^4 \text{ N/m}}{}\]

(d) work from basics

(e) \(x = \pm A/2\) \(\Rightarrow K = ? \ U = ?\)

(f) \(E = \text{no calc}\)

g) \(x = \pm A/2\) \(\Rightarrow K = ? \ U = ?\)

(h) \(E = \frac{1}{2} kA^2 = \frac{1}{2} \times 14.6 \times 0.1^2 = 0.098 \text{ J}\)

(i) \(U = \frac{1}{2} k(A)^2 = \frac{1}{4} E = 0.0245 \text{ J} = 2.45 \times 10^{-3} \text{ J}\)

\[K = E - U = \frac{3}{4} E = 0.735 \text{ J} = 7.35 \times 10^{-3} \text{ J}\]

\[\text{at end, a push where x = -A}\]

\[u=0 \quad x=0 \]

\[u\rightarrow v \quad \text{for push where x = -A}\]

\[\text{Compare effect of push on}\]
a) energy \(\frac{1}{2} kA^2 + \frac{1}{2} m v_0^2\)

b) \(v_{max} = \frac{1}{2} m v_{max}^2 \text{ (bigger)}\)

c) \(a_{max} = \text{bigger! (b) stretches more at other side! (recall}\}

\[\text{for push where x = -A, F = -k(A+4A) =}

\[\text{example}\]

\[\text{example}\]
Spider web

- $m_{spider} = 0.30$ grams
- Web negligible mass
- $f = 15$ Hz
  a) $k = ?$
  b) If $m_{insect} = 0.10$ grams with spider, freq of vibration $= ?$
  c) Show $f = \frac{f_0 \times ?}{15}$

- $m = 13 \times \left(\frac{3}{4}\right)$ Hz

Large motor on factory floor

- Floor amplitude near motor $\approx 3.0$ mm
- Frequency $10$ Hz
- Max acceleration of floor near motor?

- $\omega = \sqrt{\frac{k}{m}} \Rightarrow k = (2\pi f)^2 m$
- $a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{2\pi f}{m} A = (2\pi 10)^2 \times 3 \times 10^{-3}$
- $= 4 \times 10^2 \times 3 \times 10^{-1} = 11.8 \approx 12 \text{ m/s}^2 > g$
- Object on floor loses contact

Simple Pendulum is also SHM!

- $F = -mg \sin \theta$
  - Taylor
  - $s \approx L \sin \theta \approx \frac{s}{L}$
  - $s = L \theta$
- $\theta_0 \sin \theta \approx 0.01745$
  - $L = 0.01745$
  - $\approx 0.005\%$
- $w = \sqrt{\frac{k}{m}} = \sqrt{\frac{9}{L}}$
  - No mass for amplitude pendulum clock is lost on swing
\[ f = \sqrt{\frac{g}{l}} \]

Ex9

\( l = 37.10 \text{ cm} \) use pendulum to measure \( g \)

\( f = 0.819 \text{ Hz} \) at some place on Earth

\( g = ? \)

\( \omega^2 = 37.10 \times 10^{-2} (2 \pi 0.819)^2 = 9.824 \text{ m/s}^2 \)

11.5 Damped Harmonic Motion

\( x(t) = Ae^{-\frac{b}{2m}t} \cos(\omega t + \phi) \)

- shock absorber in car: critically or slightly underdamped
- building dampers

\( 3 \) Sunset's red

blue scattered away

11.6 Forced Oscillation

(Push at any frequency & force to move)

- Resonance: push at resonant frequency
  - no damping: amplitude builds up
  - with damping: amplitude is big

- See CISE & book pix in ppt
  - bridge p443 (break step)
  - goblet & trumpet p547
  - radio receives freq. when tuned
  - tuning forks matching \( \bigtriangledown \bigtriangledown \) show CISE

- walking with coffee
- kid on swing

molecules like atoms vibrating at SHM when object dropped

\( \omega = \text{natural frequency, resonant} \)

- CISE - gobble-gulp!
  - atoms resonate at UV freq
  - hold energy longer/dissipate as thermal easily
  - molecules resonate at IR freq

- Sky is blue, resonate at blue (shorter) & scatter
gulp & re-emit (not like glass, gulp/collide/heat)

Smog - white - diff size re-emit all colors
pollution - brown - globs of stuff absorb colors (not scatter)
Wave Motion

- Mechanical wave propagates by oscillation of matter
- Cose SHM - energy travels \( v \)
  - matter does not go far, but oscillates SHM
  - \( v_{\text{max}} = A \omega \)
  - \( f = \frac{1}{T} \uparrow \)
  - transverse
  - \( \lambda \) - longitudinal
  - compression
  - expansion
  - rarefaction

\[
y(x) = A \cos \left( \frac{2\pi}{\lambda} x \right)
\]

\[
y(x, t) = A \cos \left( \frac{2\pi}{\lambda} (x - vt) \right)
\]

\[
y(x, t) = A \cos (kx + \omega t) \quad \text{moving left}
\]

\[
F_y = m \frac{dv_y}{dt}
\]

\[
\frac{F_y}{F} = \frac{v_{yt}}{vt}
\]

\[
\lambda = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{B}{\rho}}
\]

- On a rope
  - \( v = \sqrt{\frac{T}{\mu}} \)
  - mass/length
  - \( \text{sensible} \)
  - \( \rho_x = 0 \ m \nu^2 \)

- Why? Young & Freedman
  - impulse = \( 4P \)
  - \( F_y \ up \)
  - mass increases going up
  - \( P \)

- \( \frac{F_y}{F} = \frac{v_{yt}}{vt} \)

- Why similar triangles? tension along rope?

- \( v = \sqrt{\frac{F}{\mu}} \)

- \( v = \sqrt{\frac{\text{elastic force factor}}{\text{inertia factor}}} \)

- \( \lambda \) by shake \( f \)

- Loud/soft voice travel at same speed (though loud gets further)

- Source shakes slower:
  - \( f \uparrow \Rightarrow \text{energy} \uparrow \)

- Source shakes faster:
  - \( f \uparrow \Rightarrow \text{energy} \uparrow \)

- \( \text{Im} \) phasors engineering
  - \( \text{Re} \)
Other Waves:

10. Surface Waves: 2D: surface waves 3D: earthquake

Energy transported by waves:

\[ \text{Intensity} = \frac{\text{Power}}{\text{Area}} \sim \frac{1}{r^2} \]

\[ = \frac{\text{energy/time}}{\text{area}} \quad (U = \frac{1}{2} kA^2) \]

\[ A \sim 4 \]

\[ A \sim \frac{1}{r} \]

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ k = m \omega^2 = \rho v t S (2\pi f)^2 \]

\[ I = \frac{P}{S} = \frac{E}{t S} = \frac{1}{2} kA^2 \frac{1}{t} = \frac{1}{2} \left[ \rho v t S (2\pi f)^2 \right] \frac{A^2}{t} \]

\[ = 2\pi^2 \rho v f^2 A^2 \]

\[ I \sim f^2, A^2 \]
String 1.10 m long
\[ m = 9.00 \text{g} \]

a) \( f_1 = 131 \text{ Hz} \)

\[ \frac{\lambda}{2} = l \]

\[ v = \sqrt{\frac{f}{\mu}} \quad v = \lambda f \]

\[ T = \mu v^2 \]

\[ = \frac{m}{\lambda} (\lambda f)^2 = 67.9 N \]

b) \( f_2, f_3, f_4 \)

\[ f_2 = 2f_1 = 262 \text{ Hz} \]

\[ f_3 = 3f_1 = 393 \text{ Hz} \]

\[ f_4 = 4f_1 = 524 \text{ Hz} \]

\[ \sin \theta_1 = \frac{l_1}{a} = \frac{v_1 t}{a} \quad \Rightarrow \quad \frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} \]

\[ \sin \theta_2 = \frac{l_2}{a} = \frac{v_2 t}{a} \]

11.3 Refraction

Law of refraction

\[ \text{Snell's law: } (n_1 \sin \theta_1 = n_2 \sin \theta_2) \]

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11.14 Diffraction
Standing Waves
Resonance at 1st Harmonic

\[ f_1 = \frac{v}{2\lambda} \leftarrow f_n = \frac{v}{\lambda_n} \]

\[ f_a = \frac{v}{\lambda_a} = n \left( \frac{v}{2L} \right) \]

\[ \frac{\lambda_2}{2} \cdot 2 = \lambda \]
\[ \frac{\lambda_3}{2} \cdot 3 = \lambda \]
\[ \lambda_n = \frac{2L}{n} \]

Math. why nodes there