PME5231 Final Exam 王冠宇 博一 **d903793**

(Part 1 takes 5% of your final grade. Part 2 takes 35% of your final grade.)

Part 1- For parallel-plate capacitive micro-actuators-

Question 1-1: (40%) why don't we use charge control instead of voltage control?

Ans：

(1-1) 由figure 1(green line)可知，電流持續加大時，deflection(z)也會跟著加大。電流若是繼續加大時，兩個平行電容板最後會接觸在一起。所以若使用charge control的話，無法判斷pull-in charge的值在那裡，如此一來，使用者可能會輸入太多的charge，而自己都不知道。兩平行電容板間的電流會持續存在，當兩個平行電容板接觸時，此時電流若還是繼續加大時，平行電容板會以產熱的方式來消耗持續輸入的電流。因此若使用voltage control的話，當電壓持續加大時，直到超過pull-in voltage時。此時兩個平行電容板會接觸在一起，兩者間的電位差會等於零，造成短路，figure 2(green line)。此時使用者即知兩個平行電容板以接觸在一起。故使用voltage control比charge control來的好。

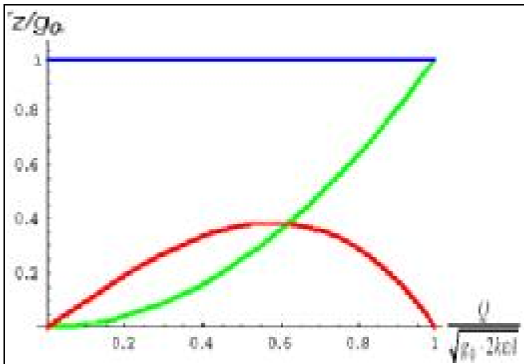


Figure 1. Plot of normalized deflection (z/g0, green curve) and voltage ($V\times\frac{1}{g_0}\sqrt{\frac{Ae}{g_0 2k}}$，red curve) vs. normalized charge ($\frac{Q}{\sqrt{q\times 2keA}}$) for a charge controlled, electrostatic parallel plate actuator.

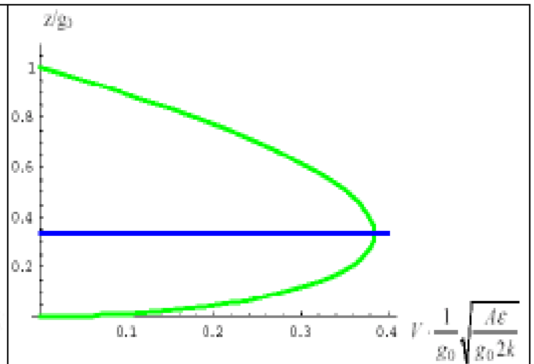


Figure 2. Graph showing normalized deflection, z/g0, as a function of normalized voltage in an electrostatic, parallel-plate actuator. There are two equilibrium deflections for each value of the voltage. The solutions corresponding to the upper branch of the graph are unstable.

Question 1-2:(60%) Please give a design to let actuating distance overcome 1/3 of nominal plate separation.

Ans：

(1-2) 理論上只要deflection大於1/3原長的話，此時兩平行電容板便會產生pill-in effect，而接觸在一起。所以設計一個mechanical stop的結構，當電壓持續增加時，deflection of cantilever beam最多也只會到達mechanical stop此結構的地方(figure 3)。

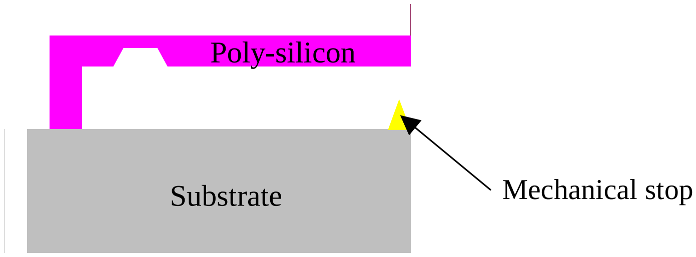


Figure 3.

Part 2-Simulation of a Tunneling Accelerometer

The side view of a MEMS tunneling accelerometer illustration with a simple feedback circuitry is shown in the following figure.

(a) The proof mass is made by a p-type (100) 200 μm-thick Si wafer with a thin Cr/Au layer as proof mass electrode. The mass of Cr can be neglected compared with Au layer mass. The proof mass electrode and proof mass have the same area of 0.54 cm^2 and 0.8 cm long.

(b) The proof mass is 28.2 milli-gram. The resonant frequency of proof mass is 100 Hz. The quality factor Q for the proof mass is 140.

(c) Unforced gap between proof mass electrode and deflection electrode is 34 μm. The tunneling tip height is exactly 31 μm.

(d) The relationship between tunneling current and tunneling gap is $I_t \propto V_B \times \exp(-\alpha_I (\Phi)^{0.5}\ x)$,where V_B is tunneling bias across the tunneling electrode gap, α_I is a constant, α_I =1.025(Å^{-1} eV$^{-0.5}$), and Φ is the effective work function of Au which is to be measured in this exam. *x* is the tunneling gap.

(e) The gravity is ignored in this exam.

(f) The op-amp is modeled as a low pass filter with high DC gain

$$A_{op} = \frac{K_{op}}{as+1} \quad \text{where } K_{op} = 10^6 \quad a = \frac{1}{20\times p}$$

Question 2-1: (5%) What is the width of the proof mass? What is the thickness of the Au layer on proof mass?

Ans：

(2-1-a) Area of the proof mass = 0.54 cm^2，Length of the proof mass = 0.8 cm，
Total proof mass = 28.2 mg. Width of the proof mass = 0.54/0.8 = <u>0.675 cm</u>
Si wafer thickness = 200 μm

(2-1-b) $r_{Au} = 19280\ kg/m^3 \qquad r_{Si} = 2330\ kg/m^3$

$$r_{Si} = \frac{M_{Si}}{V_{Si}} = \frac{M_{Si}}{0.54\times10^{-4}\times200\times10^{-6}} = 2330 \qquad M_{Si} = 2.516\times10^{-5}\ \text{kg}$$

$$r_{Au} = \frac{M_{Au}}{V_{Au}} = \frac{M_{Au}}{0.54\times10^{-4}\times h\times10^{-6}} = 19280 \qquad M_{Au} = 1.041\times10^{-6} h\ \text{kg}$$

The thickness of the Au = h

$$\text{Total proof mass} = \mathrm{M_{Au}} + \mathrm{M_{Si}} = 28.2\times10^{-6}\ \text{kg m}$$

$$\mathrm{M_{Au}} = (2.82-2.516)\times10^{-5} = 3.04\times10^{-6} = 1.041\times10^{-6}\,\text{h} \qquad \underline{\text{h} = 2.92\,m\text{m}}$$

Question 2-2: (5%) What are the spring constant and damping coefficient of the proof mass? What are the equivalent spring constant and resonant frequency around tunneling operation position (consider softening spring effect)?

Ans：

(2-2-a) Q = 140 , m = 28.2 mg , f = 100 Hz , $w_0 = 2\pi f = 200\pi$ rad/sec

$$\text{Q} = \frac{mw_0}{b} = \frac{w_0}{2a} = \frac{1}{b}\sqrt{km}\ ,\quad w_0 = \sqrt{\frac{k}{m}}\ ,\quad a = \frac{\text{b}}{2\text{m}}$$

$$b = \frac{mw_0}{\text{Q}} = \frac{28.2\times10^{-6}\times200p}{140} = 1.26\times10^{-4}$$

$$\text{spring constant k} = \text{mw}_0{}^2 = 28.2\times10^{-6}\times(200p)^2 = \underline{11.13}\ N/m$$

$$\text{damping coefficient}\ x = \frac{a}{w_0} = \frac{b}{2mw_0} = \frac{1.26\times10^{-4}}{2\times28.2\times10^{-6}\times2p\times100} = \underline{3.56\times10^{-3}}$$

(2-2-b)

$$m\ddot{x} + kx = \frac{e_0V^2}{2(s_0-x_0)^2}\left(1+2\frac{x-x_0}{s_0-x_0}\right)$$

$$\Rightarrow m\ddot{x} + \left(k - \frac{e_0V^2}{2(s_0-x_0)^3}\right)x = \frac{e_0V^2}{2(s_0-x_0)^2}\left[1-2\frac{x_0}{s_0-x_0}\right]$$

$$w_{res} = \left(\frac{k}{m} - \frac{e_0V^2}{m(s_0-x_0)^3}\right)^{\frac{1}{2}} = \left(\frac{k}{m} - \frac{2kx_0}{m(s_0-x_0)}\right)^{\frac{1}{2}} = \sqrt{\frac{\text{k}}{\text{m}}}\sqrt{1-\frac{2x_0}{s_0-x_0}}$$

$$\text{The resonant frequency}\ w_{res} = \sqrt{\frac{\text{k}}{\text{m}}}\sqrt{1-\frac{2x_0}{s_0-x_0}}$$

$$= 100(\text{Hz})\times\sqrt{1-\frac{2\times3}{34-3}} = \underline{89.8\,(\text{Hz})}$$

The voltage required for a specific static deflection (x_0)

$$kx_0 = \frac{e_0V^2}{2(s_0-x_0)} \Rightarrow \frac{e_0V^2}{(s_0-x_0)^3} = \frac{2kx_0}{(s_0-x_0)}$$

$$\text{The equivalent spring constant K} = k - \frac{e_0V^2}{2(s_0-x_0)^3} = k - \frac{kx_0}{(s_0-x_0)} = mw_{res}{}^2$$

$$= 28.2\times10^{-6}\times(2p\times89.8)^2 = \underline{8.978\ N/m}$$

Question 2-3: (20%) Download laser vibrometer measurement data ⟶ Xtun.mat (http://mx.nthu.edu.tw/~chhsliu/MEMS/Xtun.mat) and Vtip.mat (http://mx.nthu.edu.tw/~chhsliu/MEMS/Vtip.mat). These data show the tip voltage corresponding to individual tunneling gap. The data sampling rate is 10000Hz. The proof mass is oscillated at 200 Hz during measurement experiment using Laser Vibrometer. The Vtip also collects some noise. Based on these measurement data, find the values of I_o and Φ. *where* $I_t = I_0 \times \exp(-a_I \sqrt{\Phi} x)$

Ans：

(2-3) $I_t = I_0 \times \exp(-a_I \sqrt{\Phi} x)$ $\frac{V_{tip}}{R} = I_0 \times \exp(-1.025\sqrt{\Phi} x)$

$$\ln\frac{V_{tip}}{R} = \ln(I_0 \times \exp(-1.025\sqrt{\Phi} x))$$

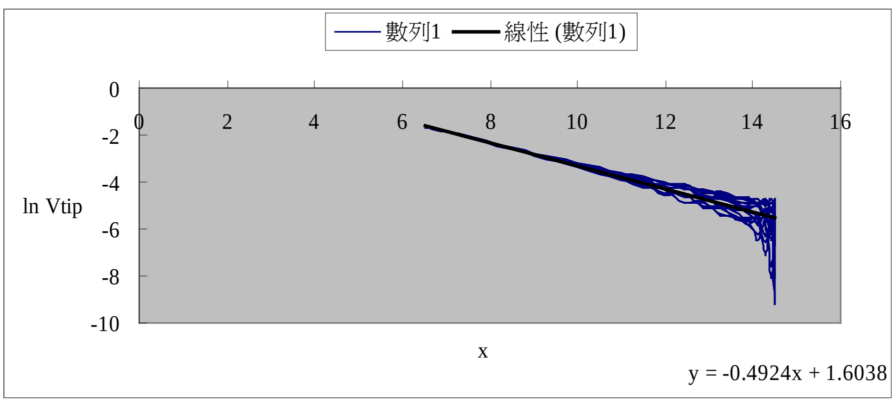
$$\ln V_{tip} - \ln R = \ln I_0 - 1.025\sqrt{\Phi}x$$

$$\ln V_{tip} - \ln(22\times10^6) = \ln I_0 - 1.025\sqrt{\Phi}x$$

$$\ln V_{tip} - 16.9 = \ln I_0 - 1.025\sqrt{\Phi}x$$

$$\ln V_{tip} = \ln I_0 + 16.9 - 1.025\sqrt{\Phi}x$$

從老師所給的V_{tip}和x，由excel畫出ln V_{tip}和x的圖形。



由圖形可得知方程式為 $y = -0.4924x + 1.6038$

相對於 $\ln V_{tip} = -1.025\sqrt{\Phi}x + \ln I_0 + 16.9$

⇨ 兩式相等以求出I₀以及Φ

$$-0.4924 = -1.025\sqrt{\Phi} \qquad \underline{\Phi = 0.23 \ \text{eV}}$$

$$1.6038 = \ln I_0 + 16.9 \qquad \underline{I_0 = 2.27\times10^{-7}} \ \text{Å}$$

⇨ $I_t = 2.27\times10^{-7} \times \exp(-0.49x)$

Question 2-4: (70%) Develop a simulink model to simulate this Micro ElectroMechanical System. (nonlinear model)

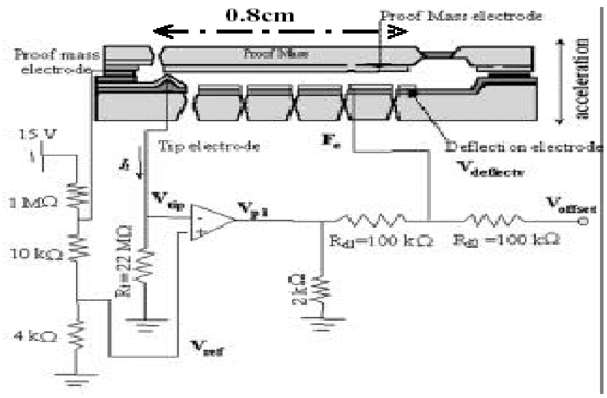
(a) Submit your simulink file.

(b) Plot Vpl and Vdeflectv vs. time for 0.1 μg external acceleration at 50 Hz.

(c) Plot Vpl and Vdeflectv vs. time for 0.1 μg external acceleration at 500 Hz.

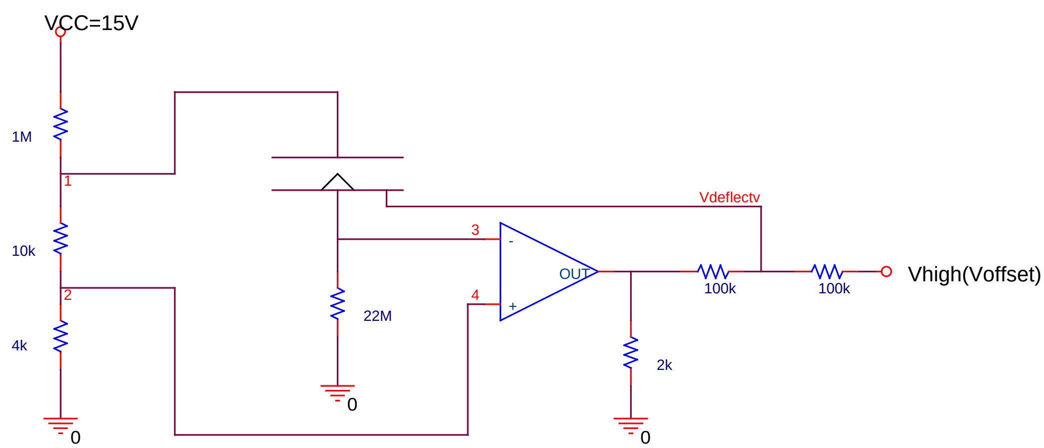
(d) Plot Vpl and Vdeflectv vs. time for 0.1g external acceleration at 50 Hz.

(e) Plot Vpl and Vdeflectv vs. time for 0.1 g external acceleration at 500 Hz.



Ans：

(a) 將 MEMS tunneling accelerometer illustration with a simple feedback circuitry。畫為下面的示意圖



By Taylor expansion at operating point, the tunneling current can be express as:

$$I_t \propto I_0 \exp(-a\sqrt{f}\,x)$$

$$I_t(x) = I_t(0) + (x - x_0)\frac{\partial I_t}{\partial x}\Big|_{x_0} + \ldots$$

The force generated by applying voltage on the electrodes is:

$$Fe = \frac{1}{2}\frac{e_0 eA}{(h+x)^2}(V_4 - V_1)^2 = \frac{1}{2}k_f(V_4 - V_1)^2 = \frac{1}{2}k_f V^2$$

$$V_1 = \frac{10k+4k}{1M+10k+4k}\times 15 = 0.207\,volt \quad \mathrm{V}_2 = \frac{4k}{1M+10k+4k}\times 15 = 0.059\,volt$$

$$\mathrm{I_t} = \frac{V_3}{R_t} = \frac{0.059}{22M} = 2.69\times 10^{-9}\ A \qquad \text{Tunneling current}$$

$$\mathbf{Q}\ \mathrm{I_t} = 2.27\times 10^{-7}\times \exp(-0.49x)$$

將 It 代入 $2.69\times 10^{-9} = 2.27\times 10^{-7}\times \exp(-0.49x)$

$$\therefore 1.185\times 10^{-2} = \exp(-0.49x) \quad \ln 1.185\times 10^{-2} = -0.49x \quad \underline{\mathrm{x} = 9.052}\ \mathring{\mathrm{A}}$$

$$\frac{0.54\times 10^{-4}\times 8.854\times 10^{-12}\times V^2}{2\times((31+0.0009)\times 10^{-6})^2} = kx = 11.13\times 2.9991\times 10^{-6}$$

$$V = 11.6 = V_{deflectv} - V_1 = V_{deflectv} - 0.207 \Rightarrow V_{deflectv} = 11.807\,volt$$

$$V_{deflectv} = 11.807 = \frac{V_{tip}+V_{offset}}{2} = \frac{V_{offset}}{2} \Rightarrow V_{offset} = \underline{22.614}\,volt$$

此 MEMS tunneling accelerometer 結構是利用 V_{offset} 來當作定位的 feedback control，所以我們可以算出大約在 9.05 Å 時，V_{offset} 的電壓約為 22.6 volt。此時若 gap 上下變動時，V_{offset} 的電壓也會跟著變動，以調整 gap 維持在 9.05Å 左右。

(b) Plot Vpl and Vdeflectv vs. time for 0.1 μg external acceleration at 50 Hz.

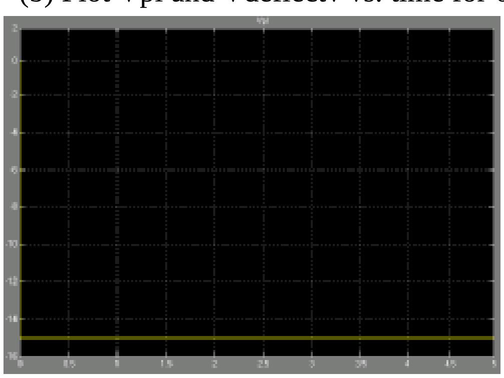


Figure 4 Vpl vs. time

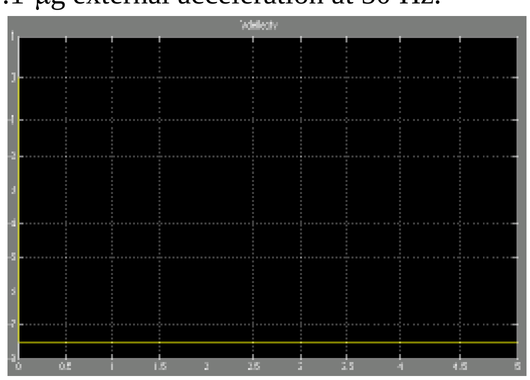


Figure 5 Vdeflectv vs. time

(c) Plot Vpl and Vdeflectv vs. time for 0.1 μg external acceleration at 500 Hz.

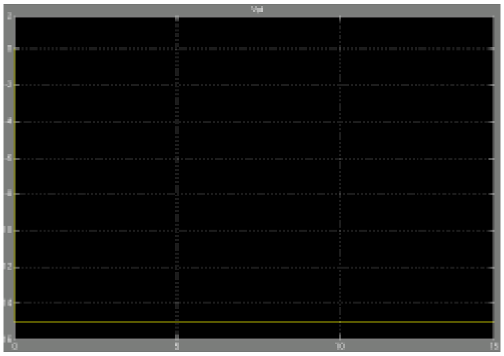


Figure 6 Vpl vs. time

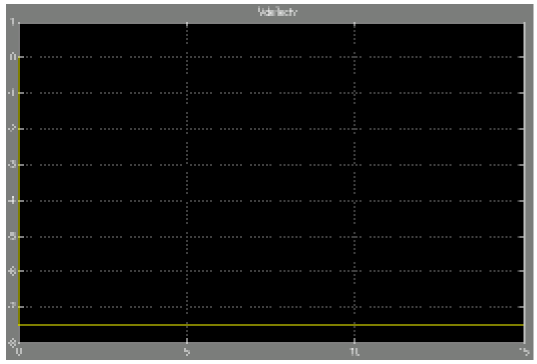


Figure 7 Vdeflectv vs. time

(d) Plot Vpl and Vdeflectv vs. time for 0.1g external acceleration at 50 Hz.

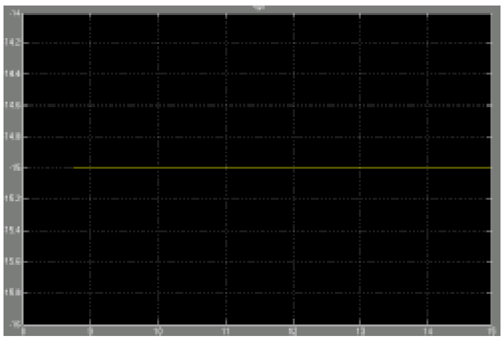


Figure 8 Vpl vs. time

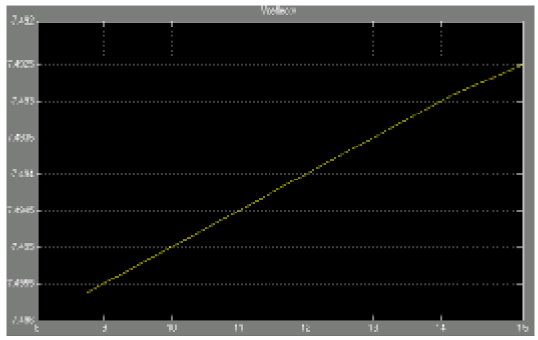


Figure 9 Vdeflectv vs. time

(e) Plot Vpl and Vdeflectv vs. time for 0.1 g external acceleration at 500 Hz.

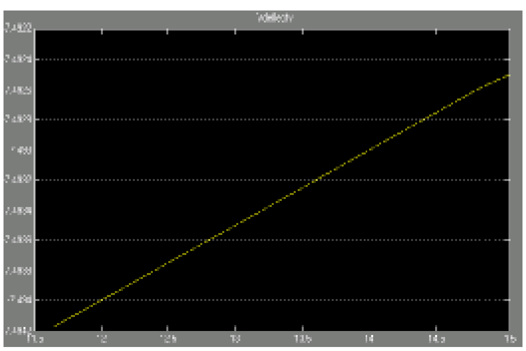


Figure 10 Vpl vs. time

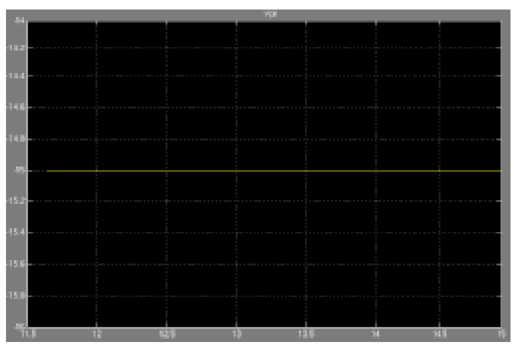


Figure 11 Vdeflectv vs. time

模擬結果與心得：

此次模擬的結果是有問題的，因為在模擬的過程中，我覺得 Fe 的次方應約為 10^{-5}、x 的次方應約為 10^{-6}、It 的次方應約為 10^{-9}、Vtip 的次方應約為 10^{-2} 左右，如此一來，(V2－Vtip)的值這會有正有負，再經過 op-amp gain 以及 saturation 後，Vpl 的值便可從+15~－15 之間的變動產生。之後產生的 Vdeflectv 的圖也是和 Vpl 的圖類似，這樣的結果我覺得才是理想的(可參考 Fig.12、13)。

下面這兩個圖形在 0.1 μg external acceleration at 500 Hz，我作 debug 時，發現若將 simulink 內 It(s)/gap(s)的 transfer function 的 gain 從 2.27×10^{-7} 變成 2.27×10^{-13} 之後所算出 Vpl and Vdeflectv vs. time 的圖形，這種圖形才是符合我所預期的圖形。

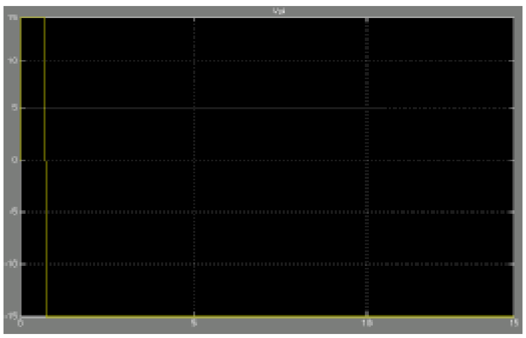


Figure 12 Vpl vs. time

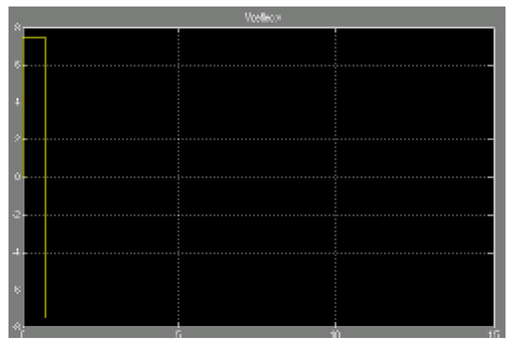


Figure 13 Vdeflectv vs. time

我模擬出來的結果 Fe 和 x 的次方是有滿足我的想法，主要是因為 30000(Å)－x(Å)的值本來應該只有幾個 Å 到幾十個 Å 左右而且。可是模擬出來的結果卻達到幾萬個 Å，與我的想法不合。導致之後的 It 只到 10^{-3} 次方，而不是到 10^{-9} 次方。所以模擬出的結果和我分析的結果有些許的差異。

我覺得原因在於：

1.對於 X(s)/F(s)我是使用線性的 transfer function，理想上應該是使用非線性的 transfer function 來進行模擬。

2.有關 Voffset 的 ramp 斜率也會影響 Vpl 和 Vdeflectv 的圖。

3.加速度的頻率變化也會影響其 Vpl 和 Vdeflectv 圖的模擬結果。

4.使用不同的 solver options 也會影響所模擬出來的圖。

這次的期末報告，雖然很辛苦，一大多不是很懂的東西，但是我作的很開心。因為從中學到了從前許多未知的觀念。從一開始什麼都不太懂，慢慢建 simulink 的 model、模擬、分析，到最後能找出一些 key point，而且也能預期模擬出來的圖形要長什麼形狀、次方大約多少等等。這期末報告完全符合這門課的名稱”微機電設計與分析模擬”。

從前未作過類似的期末報告，所以感覺很新鮮也很有趣，雖然很辛苦而模擬結果也不是我預期的。可是，一切還是值得的。